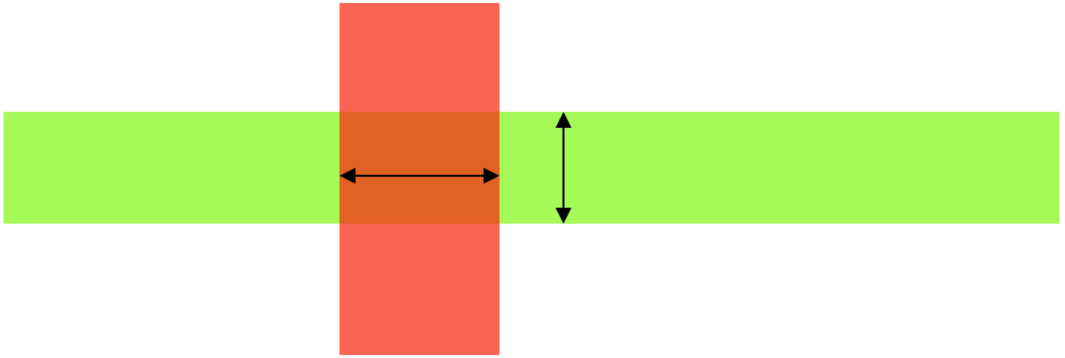
EE 330
Lecture 4

- Yield
- Statistics Review

Review from last lecture:

Feature Size

Feature size is the minimum lateral feature size that can be **reliably** manufactured



Often given as either feature size or pitch

Minimum feature size often identical for different features

Extremely challenging to decrease minimum feature size in a new process

What is meant by “reliably”

Yield is acceptable if circuit performs as designed even when a very large number of these features are made

If P is the probability that a feature is good

n is the number of uncorrelated features on an IC

Y is the yield

$$Y = P^n$$

$$P = e^{\frac{\log_e Y}{n}}$$

Example: How reliable must a feature be?

n=5E3

Y=0.9

$$P = e^{\frac{\log_e Y}{n}} = e^{\frac{\log_e 0.9}{5E3}}$$ **=0.999979**

But is n=5000 large enough ? is Y large enough?

More realistically n=5E9 (or even 5E10)

Consider n=5E9

$$P = e^{\frac{\log_e Y}{n}} = e^{\frac{\log_e 0.9}{5E9}}$$ **=0.999999999979**

20 parts in a trillion or size of a piece of sheetrock relative to area of Iowa

Extremely high reliability must be achieved in all processing steps to obtain acceptable yields in state of the art processes

Feature Size

- Typically minimum length of a transistor
- Often also minimum width or spacing of a metal interconnect (wire)
- Point of “bragging” by foundries
  - Drawn length and actual length differ
- Often specified in terms of pitch
  - Pitch is sum of feature size and spacing of same feature
  - Pitch approximately equal to twice minimum feature size

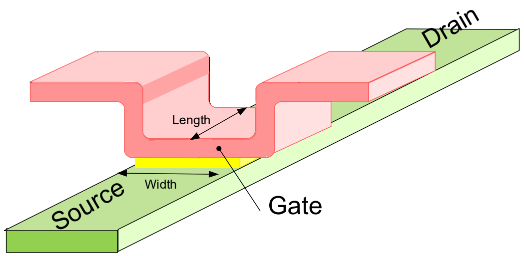
Feature Size Evolution

Mid 70's	25μ
2005	90nm
2010	20nm
2020	7nm

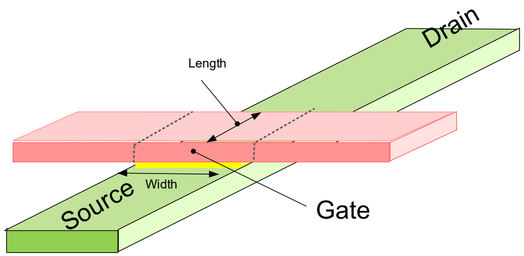
$$1\mu = 10^3\, nm = 10^{-6}\, m = 10^4\, \mathring{A}$$

Review from last lecture:

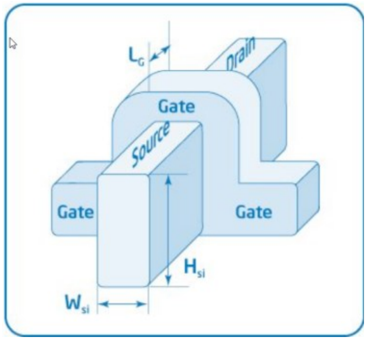
Field Effect Transistors



Planar
MOSFET
(LOCOS)



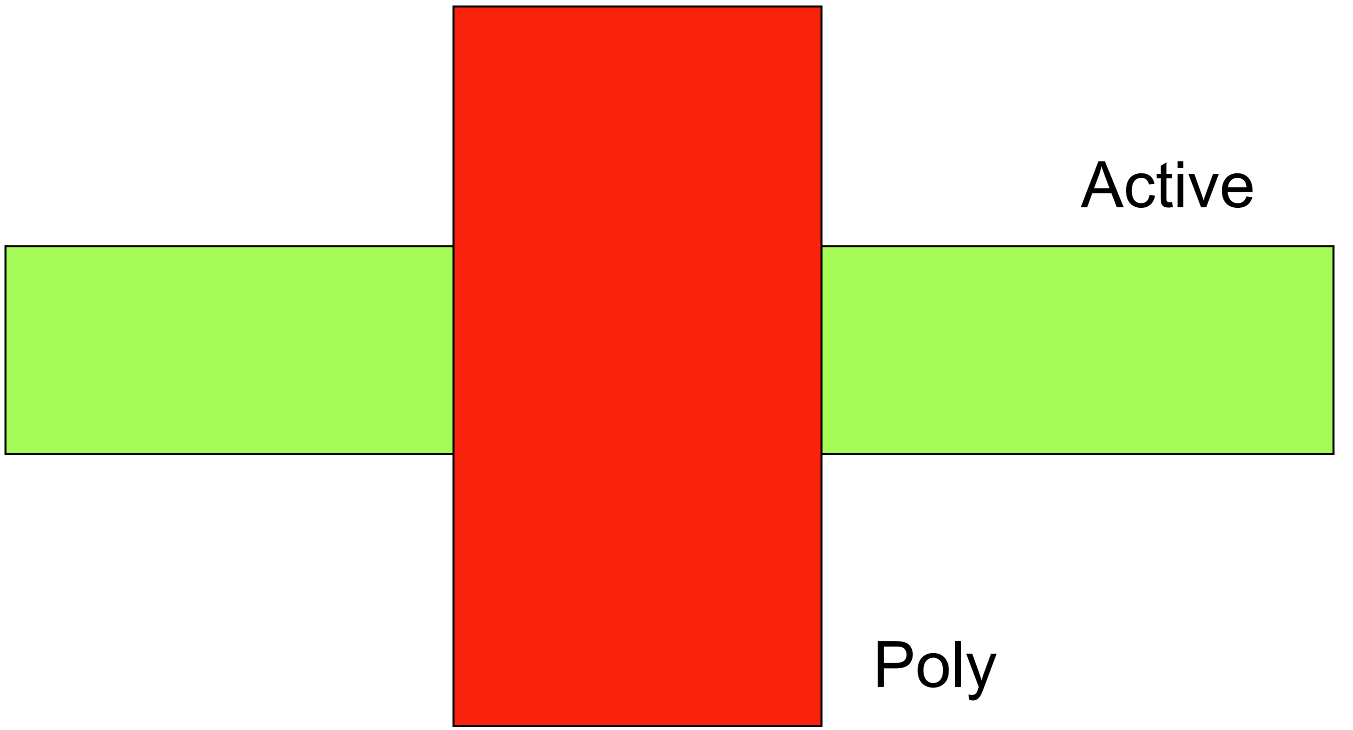
Planar
MOSFET
(STI)



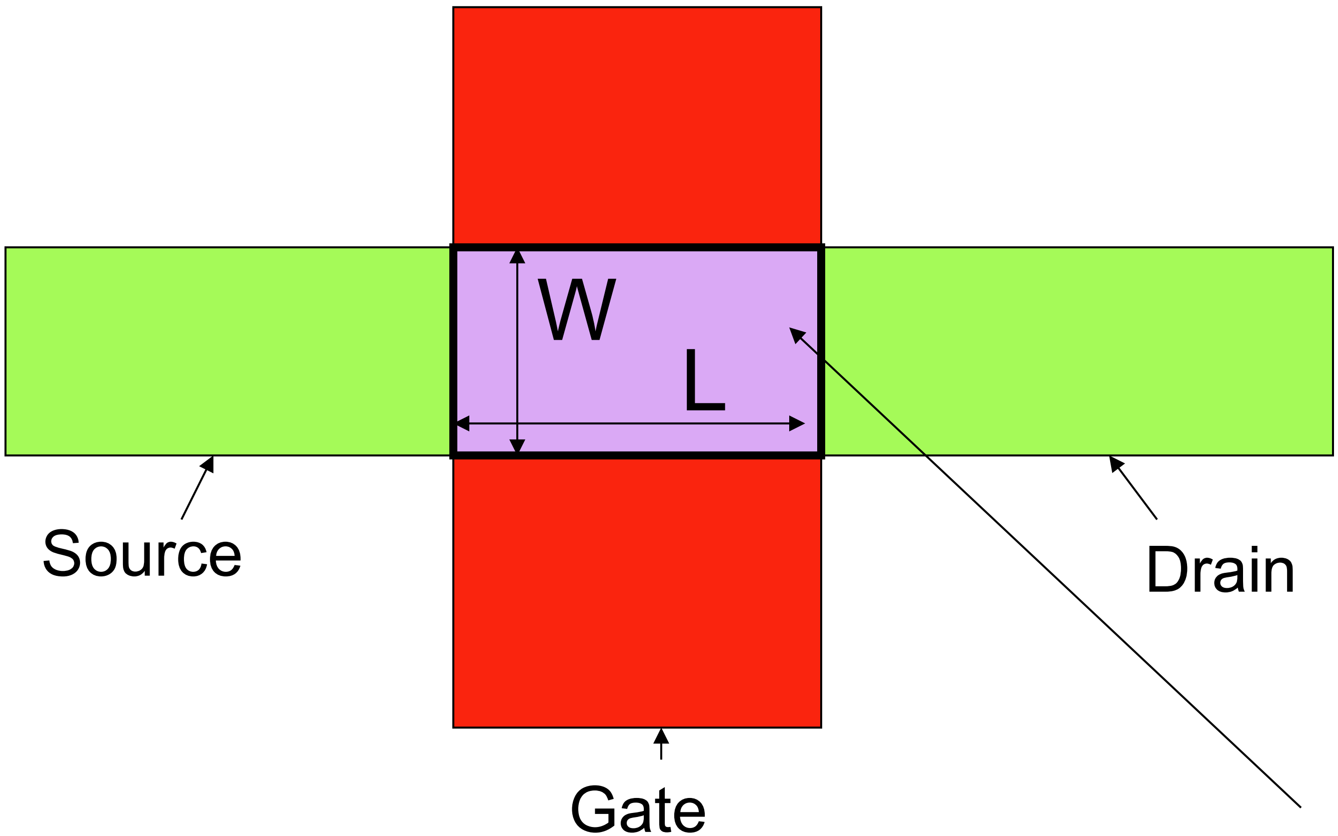
FinFET
Tri-Gate

Dielectric not shown

Planar MOS Transistor



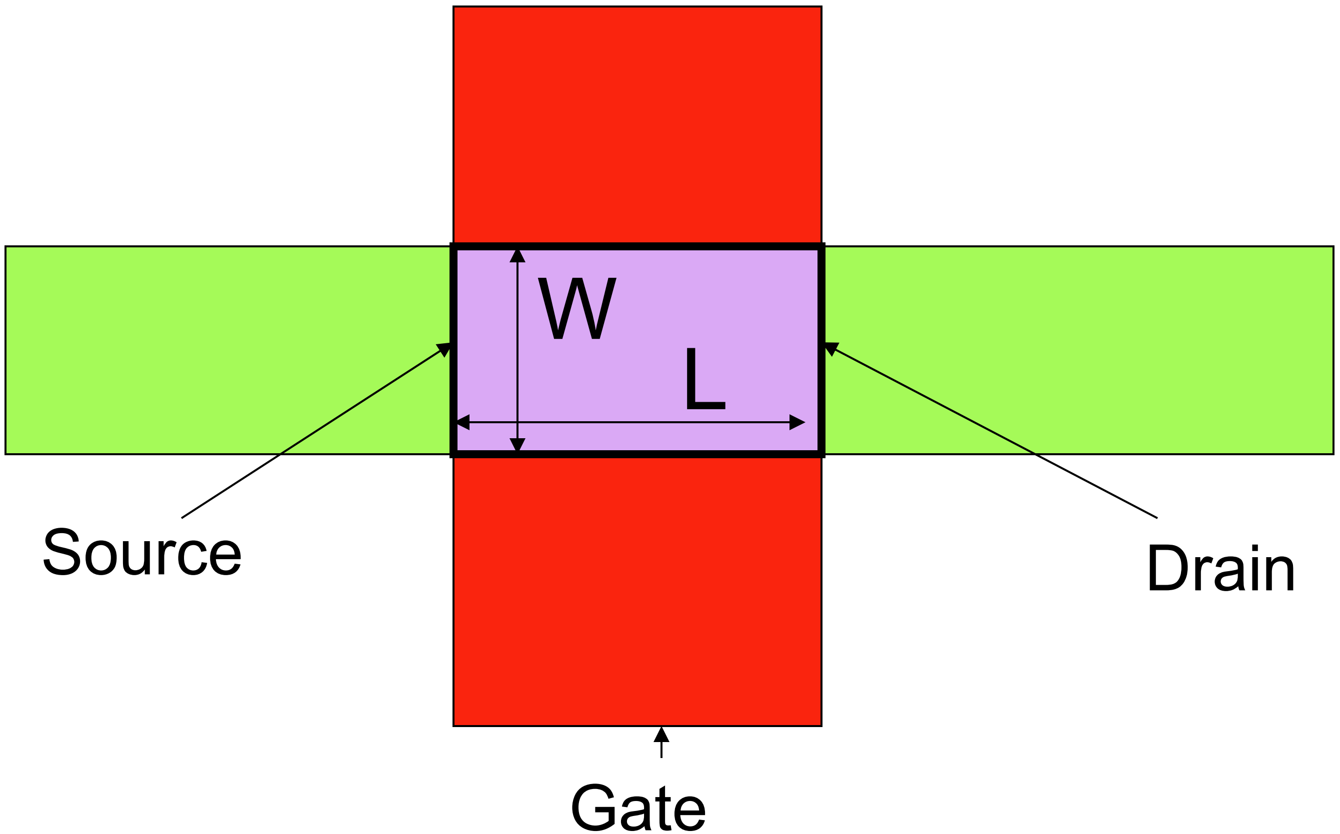
Planar MOS Transistor



Drawn Length and Width Shown

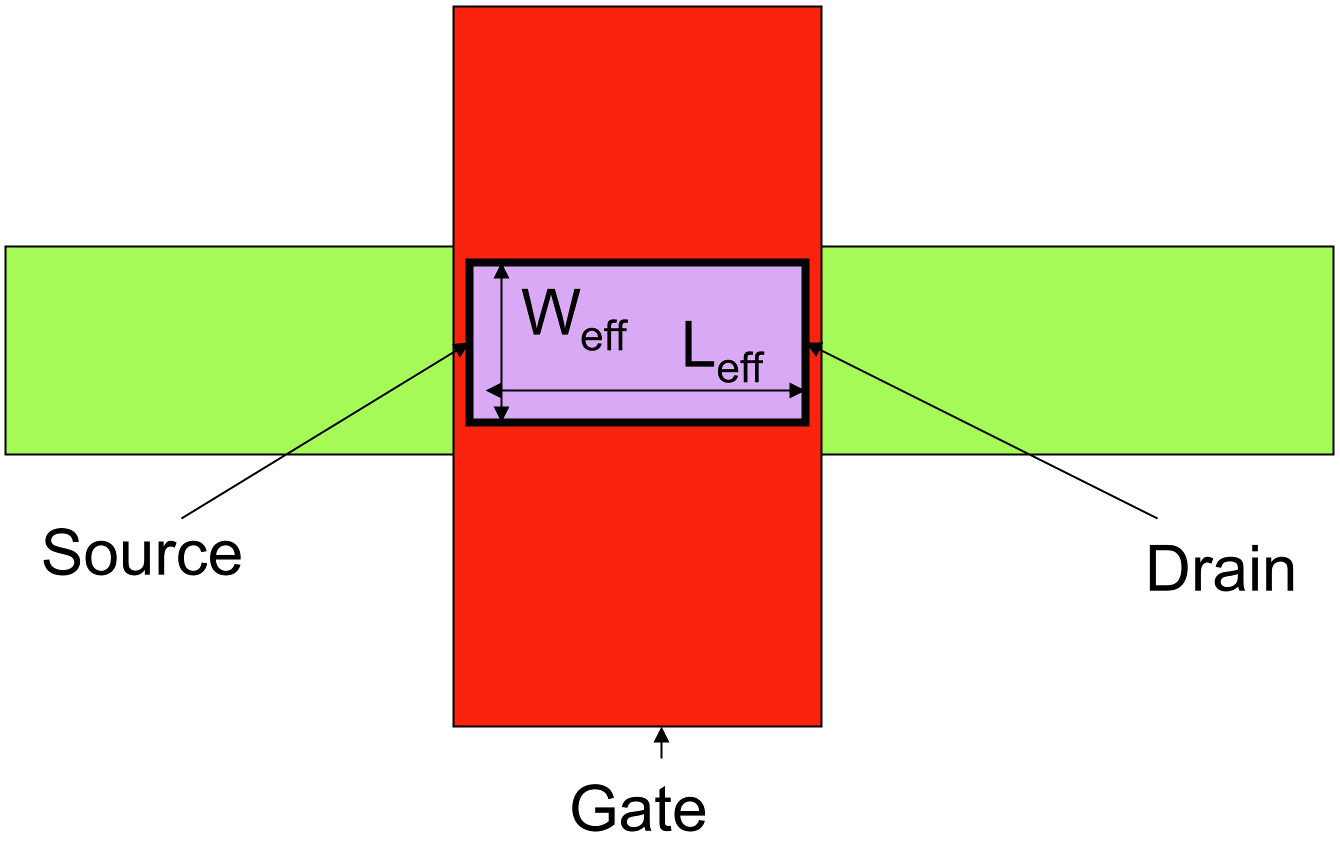
Henceforth, will assume planar devices unless specified to the contrary

MOS Transistor



Actual Drain and Source at Edges of Channel

MOS Transistor



Effective Width and Length Generally Smaller than Drawn Width and Length

Stay Safe and Stay Healthy !

Device and Die Costs

Characterize the high-volume incremental costs of manufacturing integrated circuits

Example: Assume manufacturing cost of an 8” wafer in a 0.25μ process is $800

Determine the number of minimum-sized transistors that can be fabricated on this wafer and the cost per transistor. Neglect spacing and interconnect.

Solution:

$$n_{trans} \cong \frac{A_{wafer}}{A_{trans}} = \frac{\pi(4in)^2}{(0.25\mu)^2} = 5.2E11$$

(520 Billion!)

(Trillion, Tera …10^{12})

$$C_{trans} = \frac{C_{wafer}}{n_{trans}} = \frac{\$800}{5.2E11} = \$15.4E-9$$

Note: the device count may be decreased by a factor of 10 or more if Interconnect and spacing is included but even with this decrease, the cost per transistor is still very low!

Device and Die Costs

At $800/8" wafer, it can be easily shown that:

$$C_{per\ unit\ area} \cong \$2.5 / cm^2$$

Example: If the die area of the 741 op amp is 1.8mm^2 (including bonding pads), determine the cost of the silicon needed to fabricate this op amp

$$C_{741} = \$2.5 / cm^2 \bullet \left(1.8mm^2\right) \cong \$.05$$

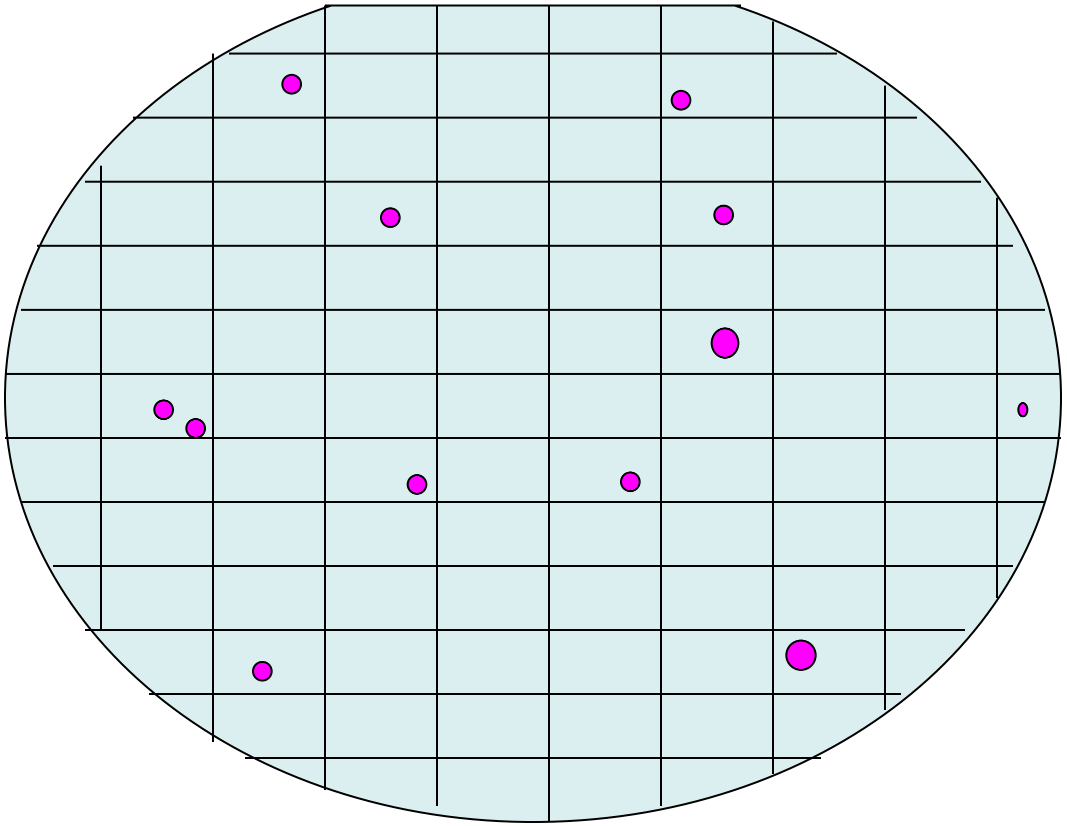
Actual integrated op amp will be dramatically less if bonding pads are not needed

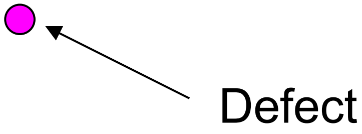
Physical Characteristics of Key Semiconductor Materials

Silicon:	Average Atom Spacing	$2.7\,\overset{o}{A}$
	Lattice Constant	$5.4\,\overset{o}{A}$
S_iO_2	Average Atom Spacing	$3.5\,\overset{o}{A}$
	Breakdown Voltage	$5 \text{ to } 10\text{MV/cm} = 5 \text{ to} 10\text{mV/} \overset{0}{A}$
Air		20KV/cm

Physical size of atoms and molecules place fundamental limit on conventional scaling approaches

Defects in a Wafer





- Dust particles and other undesirable processes cause defects
- Defects in manufacturing cause yield loss

Yield Issues and Models

- Defects in processing cause yield loss
- The probability of a defect causing a circuit failure increases with die area
- The circuit failures associated with these defects are termed **Hard Faults**
- This is the major factor limiting the size of die in integrated circuits
- Wafer scale integration has been a “gleam in the eye” of designers for 3 decades but the defect problem continues to limit the viability of such approaches
- Several different models have been proposed to model the hard faults

Yield Issues and Models

- Parametric variations in a process can also cause circuit failure or cause circuits to not meet desired performance specifications (this is of particular concern in analog and mixed-signal circuits)
- The circuits failures associated with these parametric variations are termed **Soft Faults**
- Increases in area, judicious layout and routing, and clever circuit design techniques can reduce the effects of soft faults

Hard Fault Model

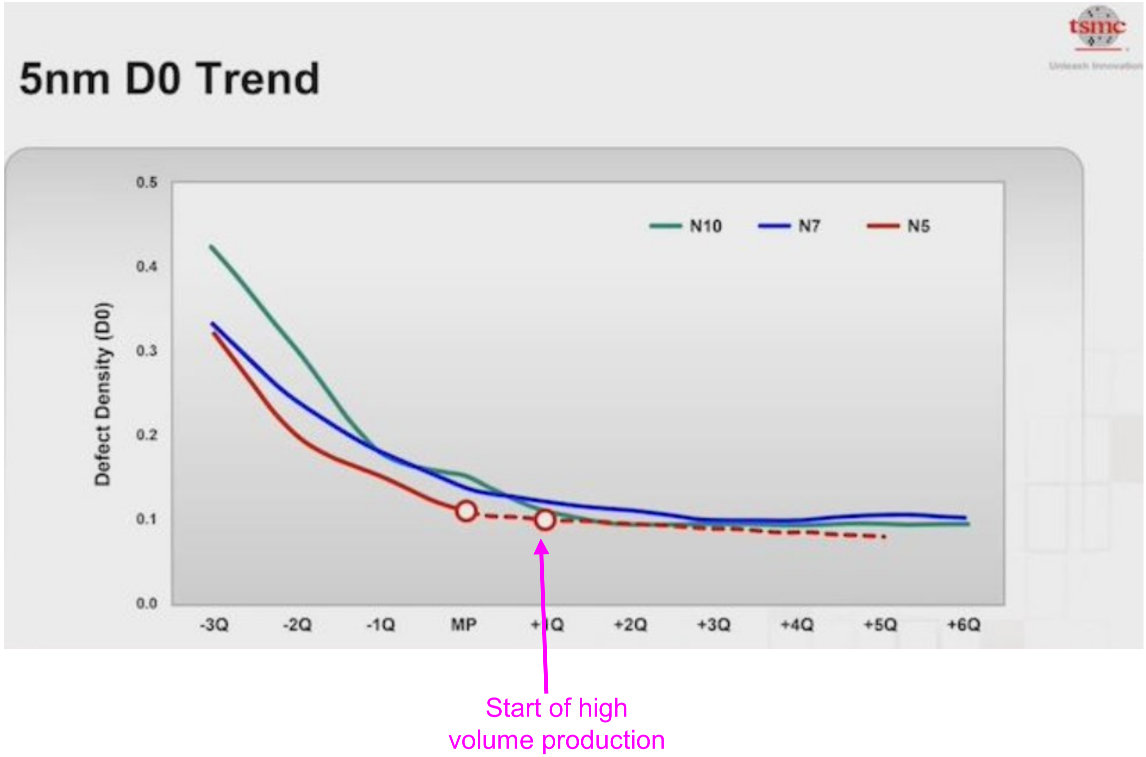
$$Y_H = e^{-Ad}$$

Y_H is the probability that the die does not have a hard fault
A is the die area
d is the defect density
(for some older processes, typically $1cm^{-2} < d < 2cm^{-2}$)
for some newer processes, typically $0.1cm^{-2}<d<1cm^{-2}$)

Industry often closely guards the value of d for their process

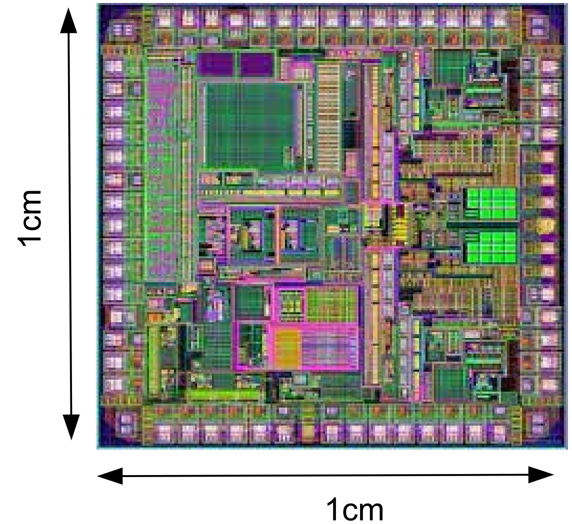
Other models, which may be better, have the same general functional form

Some processes have d under 0.1cm^{-2}



- Aug 2020 article
- Defect density in per cm^2
- Smaller processes even have better defect density!!
- Note continued reduction predicted as process matures

Example:



Determine the hard yield of a die of area $1cm^2$ if the defect density is $1.5cm^{-2}$

$$Y_H = e^{-Ad}$$

$A=1cm^2$
$d=1.5cm^{-2}$

$$Y_H = e^{-1 \bullet 1.5} = 0.22$$

How good must the defect density be if we must obtain a 95% yield for the $1cm^2$ die?

$A=1cm^2$
$Y_H=0.95$

$$0.95 = e^{-1 \bullet d} \Rightarrow d=-\ln(0.95) \Rightarrow d=0.05cm^{-2}$$

Soft Fault Model

Soft fault models often dependent upon design and application

Often the standard deviation of a parameter is dependent upon the reciprocal of the square root of the parameter sensitive area

$$\sigma = \frac{\rho}{\sqrt{A_k}}$$

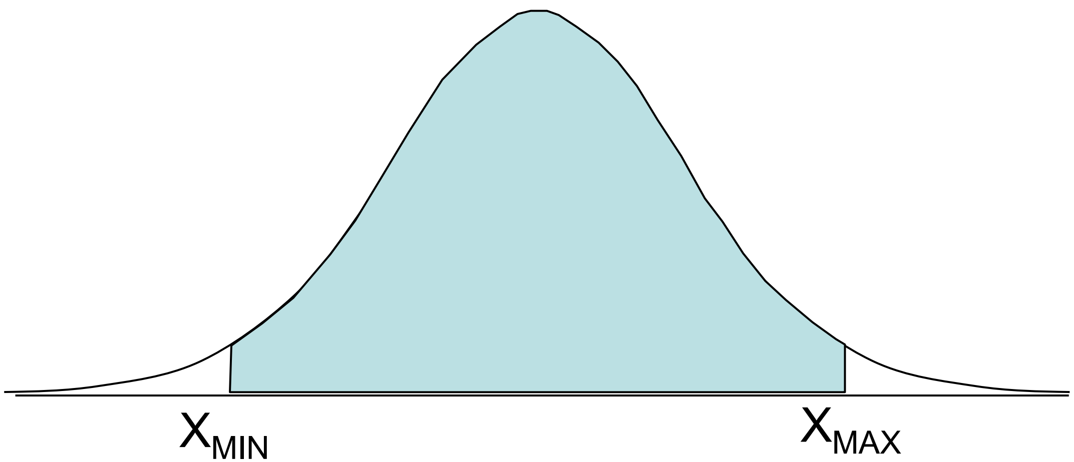
ρ is a constant dependent upon the architecture and the process

A_k is the area of the parameter sensitive area

Soft Fault Model

$$P_{SOFT} = \int_{X_{MIN}}^{X_{MAX}} f(x)\,dx$$

P_{SOFT} is the soft fault yield
f(x) is the probability density function of the parameter of interest
X_{MIN} and X_{MAX} define the acceptable range of the parameter of interest



Some circuits may have several parameters that must meet performance requirements

Soft Fault Model

If there are k parameters that must meet parametric performance requirements and if the random variables characterizing these parameters are uncorrelated, then the soft yield is given by

$$Y_S = \prod_{j=1}^{k} P_{SOFT_j}$$

Overall Yield

If both hard and soft faults affect the yield of a circuit, the overall yield is given by the expression

$$Y = Y_H Y_S$$

Cost Per Good Die

The manufacturing costs per good die is given by

$$C_{Good} = \frac{C_{FabDie}}{Y}$$

where C_{FabDie} is the manufacturing costs of a fab die and Y is the yield

There are other costs that must ultimately be included such as testing costs, engineering costs, packaging costs, etc.

Example: Assume a die has no soft fault vulnerability, a die area of $1cm^2$ and a process has a defect density of $1.5cm^{-2}$

a) Determine the hard yield
b) Determine the manufacturing cost per good die if 8” wafers are used and if the cost of the wafers is $1200

Solution

a)

$$Y_H = e^{-Ad}$$

$$Y = e^{-1cm^2 \bullet 1.5cm^{-2}} = 0.22$$

b)

$$C_{Good} = \frac{C_{FabDie}}{Y}$$

$$C_{FabDie} = \frac{C_{Wafer}}{A_{Wafer}} A_{Die}$$

$$C_{FabDie} = \frac{\$1200}{\pi(4in)^2} 1cm^2 = \$3.82$$

$$C_{Good} = \frac{\$3.82}{0.22} = \$17.37$$

Do you like statistics ?

Statistics are Real!

Statistics govern what really happens throughout much of the engineering field!

Statistics are your Friend !!!!

You might as well know what will happen since statistics characterize what WILL happen in the presence of variability in many processes !

Statistics Review

Assume x is a random variable of interest

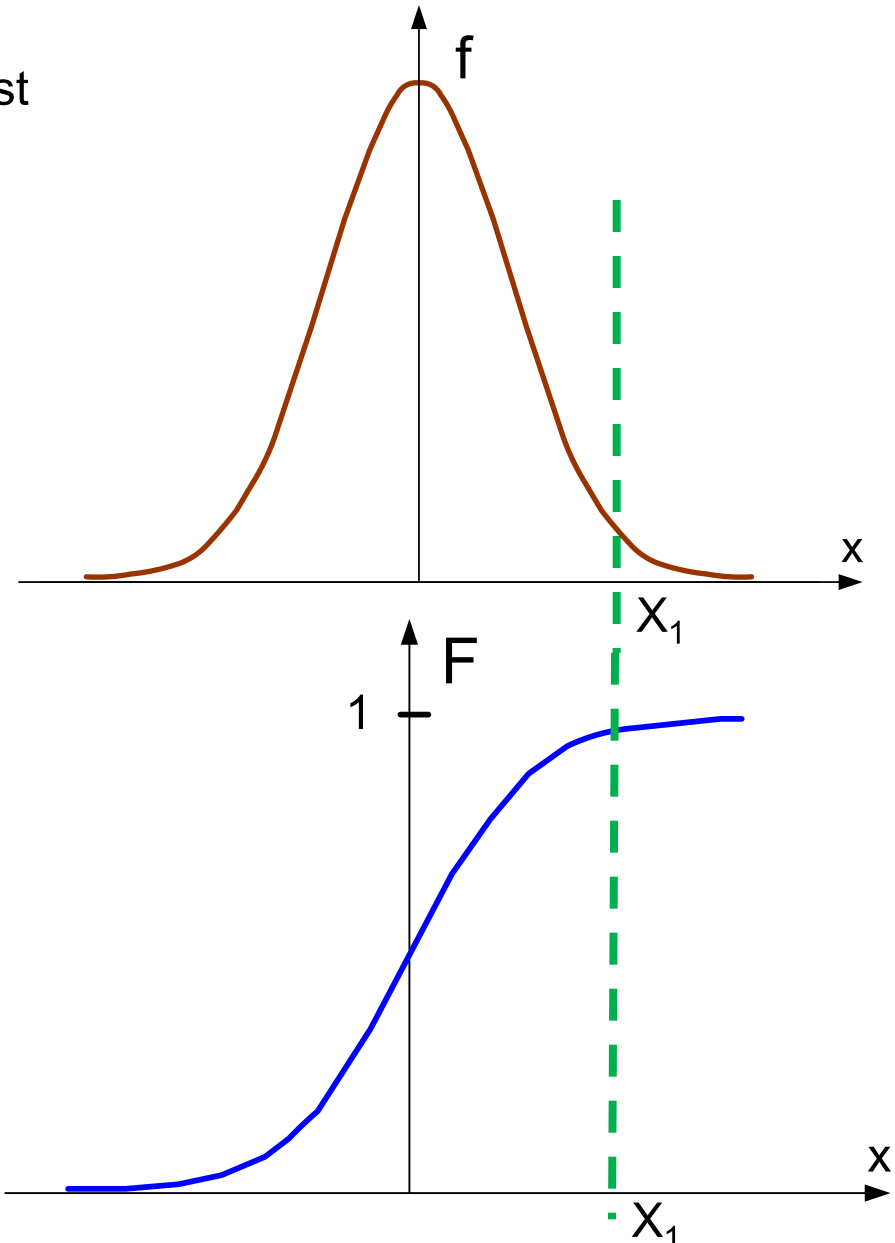
f(x) = Probability Density Function for x

$$\int_{x=-\infty}^{\infty} f(x)\,dx = 1$$

F(x) = Cumulative Density Function for x

$$F(X_1) = \int_{x=-\infty}^{X_1} f(x)\,dx$$

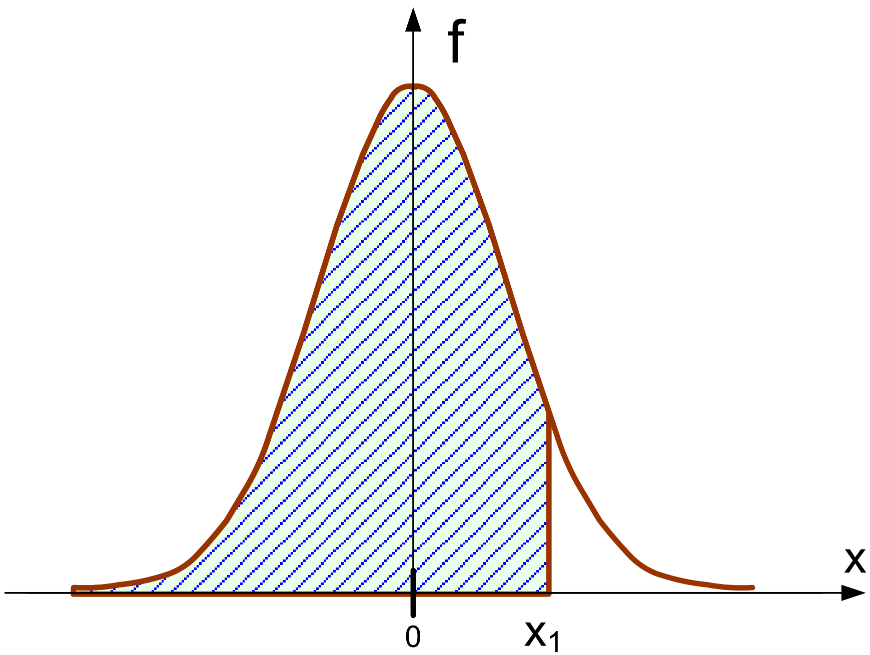
$$0 \le F(x) \le 1 \qquad \frac{\partial F(x)}{\partial x} \ge 0$$

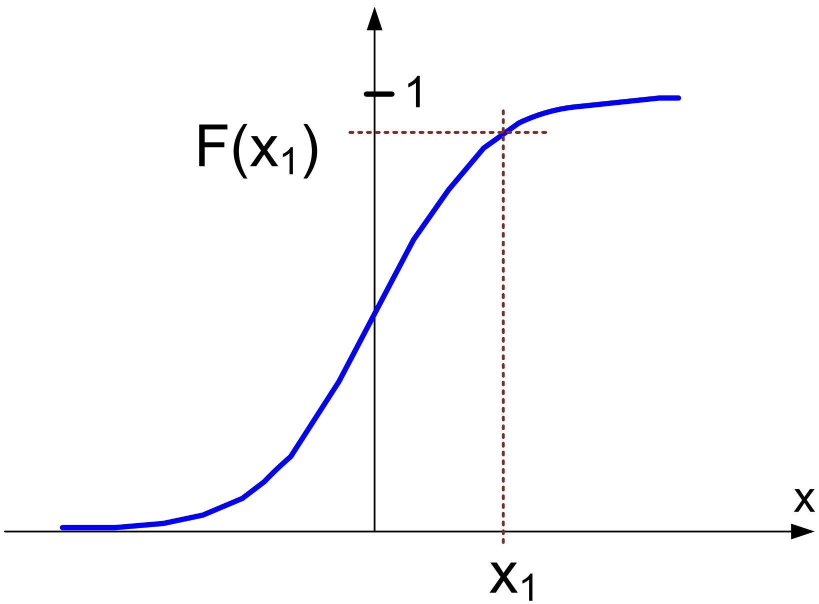


Statistics Review

f(x) = Probability Density Function for x

F(x) = Cumulative Density Function for x





$$P\{x \le x_1\} = \int_{x=-\infty}^{x_1} f(x)\,dx$$

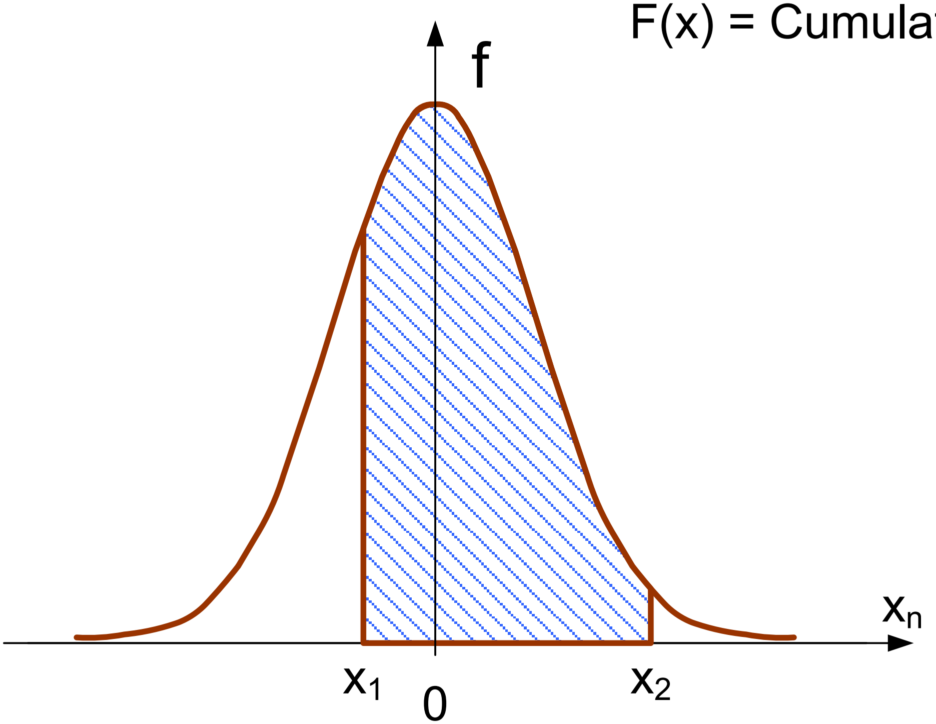
$$P\{x \le X_1\} = F(X_1)$$

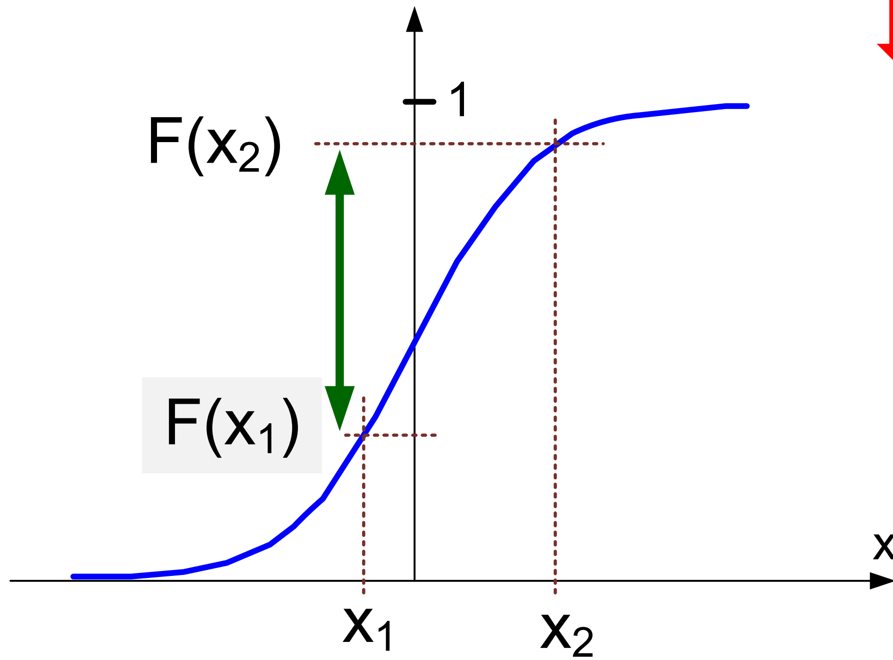
$$F(x_1) = \int_{x=-\infty}^{x_1} f(x)\,dx$$

Statistics Review

f(x) = Probability Density Function for x

F(x) = Cumulative Density Function for x

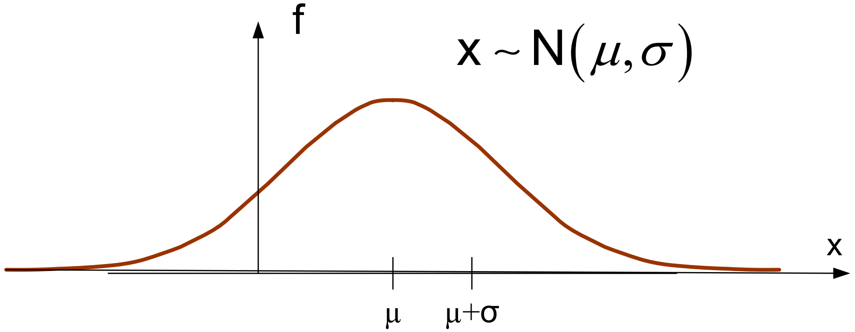




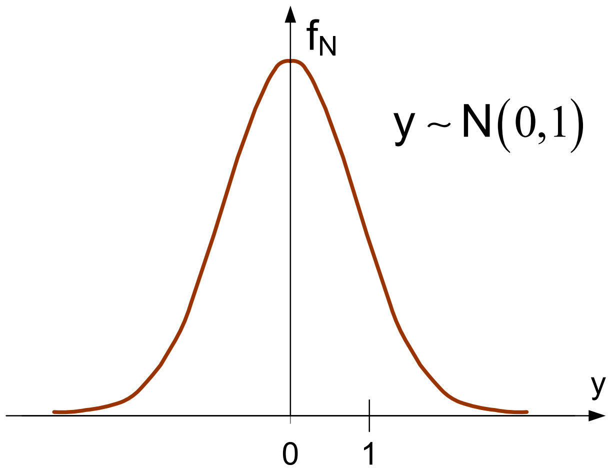
$$P\{X_1 \le x \le X_2\} = \int_{X_1}^{X_2} f(x)\,dx$$

$$P\{X_1 \le x \le X_2\} = F(X_2) - F(X_1)$$

Statistics Review



$x \sim N(\mu, \sigma)$



$y \sim N(0,1)$

$$\int_{x=-\infty}^{\infty} f(x)\, dx = 1$$

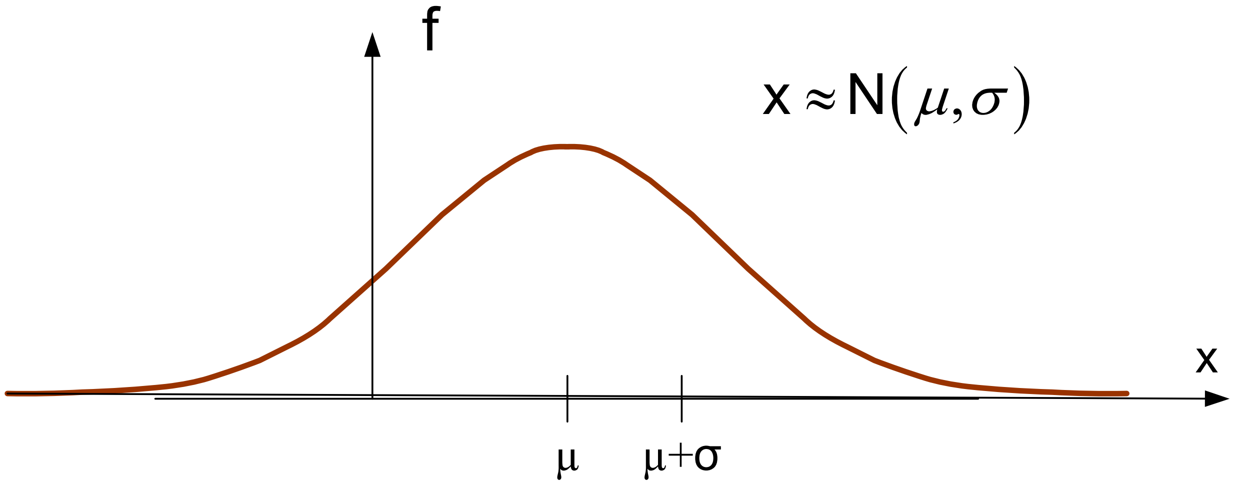
$$y = \frac{x-\mu}{\sigma}$$

$$\int_{y=-\infty}^{\infty} f_N(y)\, dy = 1$$

Theorem 1: If the random variable x is normally distributed with mean μ and standard deviation σ, then $y = \frac{x-\mu}{\sigma}$ is also a random variable that is normally distributed with mean 0 and standard deviation of 1.

(Normal Distribution and Gaussian Distribution are the same)

Statistics Review

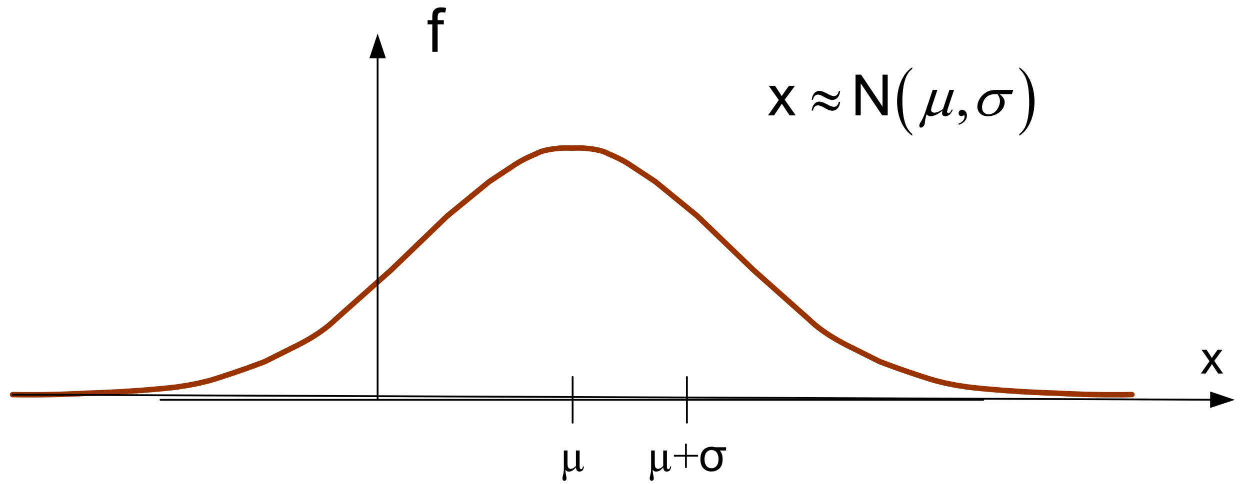


The random part of many parameters of microelectronic circuits is often assumed to be Normally distributed and experimental observations confirm that this assumption provides close agreement between theoretical and experimental results

The mapping $y = \dfrac{x-\mu}{\sigma}$ is often used to simplify the statistical characterization of the random parameters in microelectronic circuits

x generally is dimensioned, y is dimensionless

Statistics Review



Example:

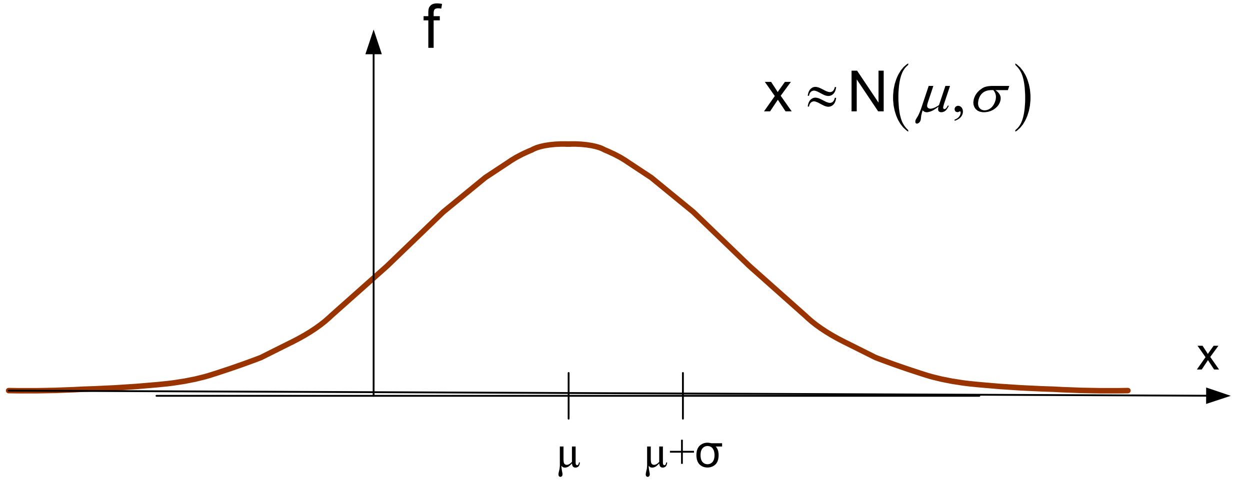
x might be the frequency of oscillation of a ring oscillator used for a clock in a crystal-less digital circuit, x Gaussian (Normal)

Dimensions of x : Hz
Maybe μ=550 MHz σ=50 MHz

$$y = \frac{x-\mu}{\sigma}$$ is dimensionless with $\mu_y=0$ $\sigma_y=1$

y: N(0,1)

Statistics Review



$x \approx N(\mu,\sigma)$

Example:

x might be the offset voltage of an op amp, x Gaussian (Normal)

Dimensions of x : Volts
Typically μ=0V σ=10 mV

$$y = \frac{x-\mu}{\sigma}$$

is dimensionless with $\mu_y=0$ $\sigma_y=1$

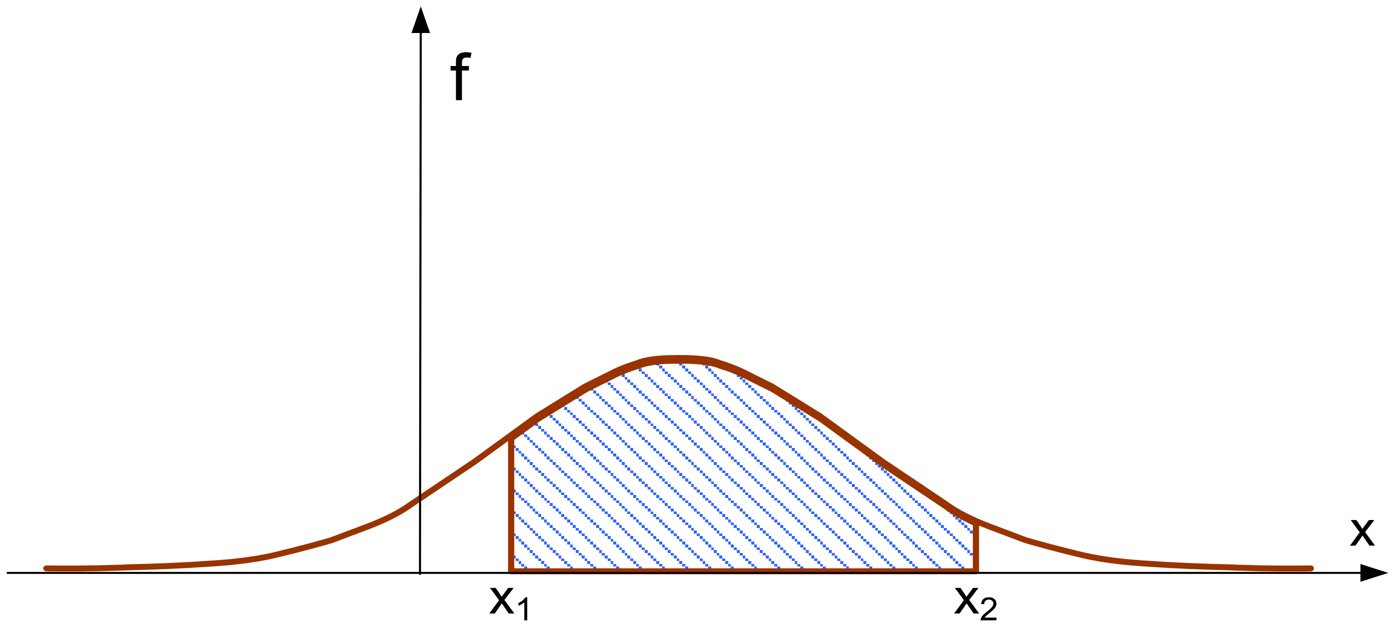
y: N(0,1)

Background Information

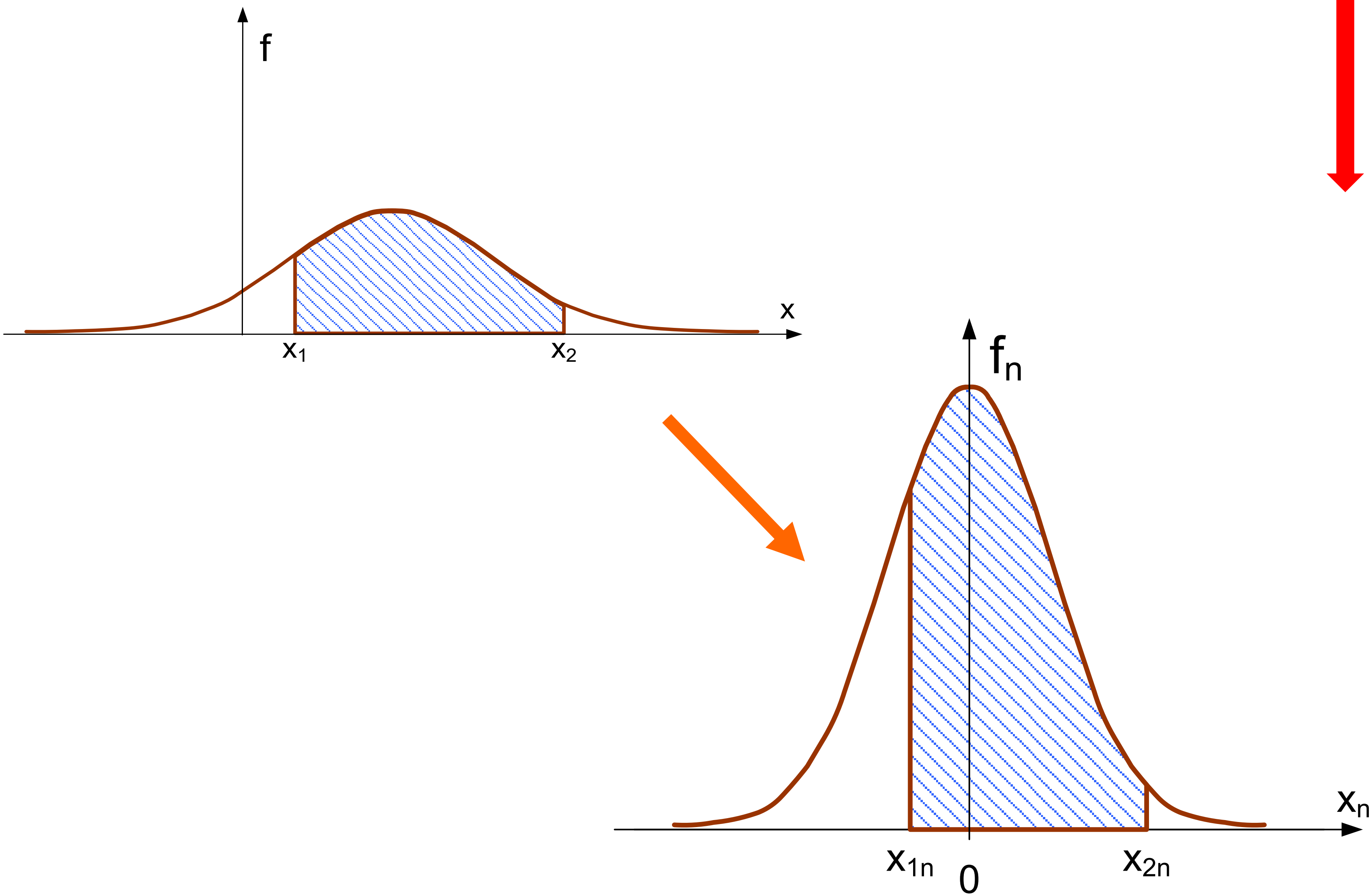
Theorem 2: If x is a Normal (Gaussian) random variable with mean μ and standard deviation σ, then the probability that x is between x_1 and x_2 is given by

$$p = \int_{x_1}^{x_2} f(x)\,dx = \int_{x_{1n}}^{x_{2n}} f_n(x)\,dx \quad \text{where} \quad x_{1n} = \frac{x_1 - \mu}{\sigma} \quad \text{and} \quad x_{2n} = \frac{x_2 - \mu}{\sigma}$$

and where $f_n(x)$ is N(0,1)



Background Information

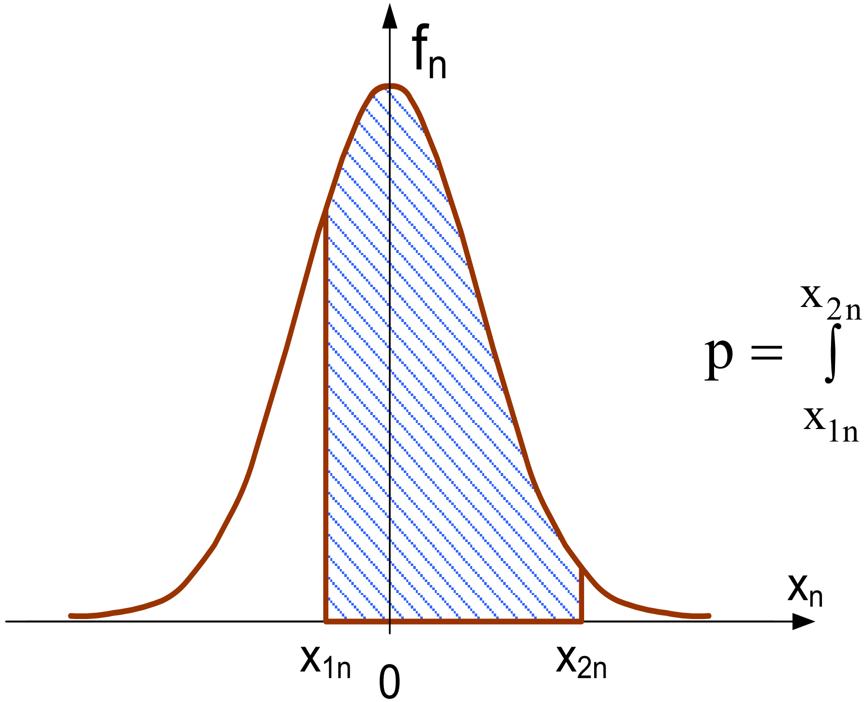


Background Information

Observation: The probability that the N(0,1) random variable x_n satisfies the relationship $x_{1n}<x_n<x_{2n}$ is also given by

$$p = F_n(x_{2n}) - F_n(x_{1n})$$

where $F_n(x)$ is the CDF of x_n.



$$p = \int_{x_{1n}}^{x_{2n}} f_n(x)\,dx$$

Since the N(0,1) distribution is symmetric around 0, p can also be expressed as

$$p = F_n(x_{2n}) - (1 - F_n(-x_{1n}))$$

Background Information

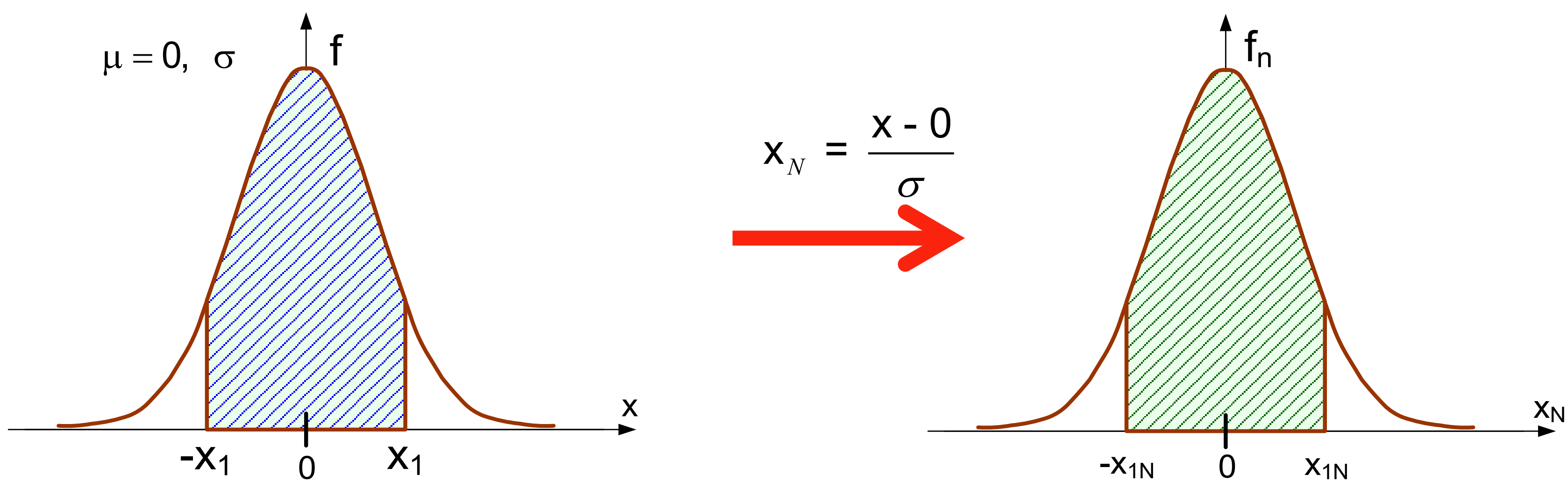
Observation: In many electronic circuits, a random variable of interest, x, is 0 mean Gaussian and the probabilities of interest are characterized by a region defined by the magnitude of the random variable (i.e. $-x_1 < x < x_1$).

In these cases, if we define $x_N = \frac{x - 0}{\sigma}$ then x_N is N(0,1) and

$$p\left(-x_1 < x < x_1\right) = \int_{-x_1}^{x_1} f(x)\,dx = \int_{-x_{1n}}^{x_{1n}} f_n(x)\,dx = F_n\left(x_{1n}\right) - F_n\left(-x_{1n}\right)$$

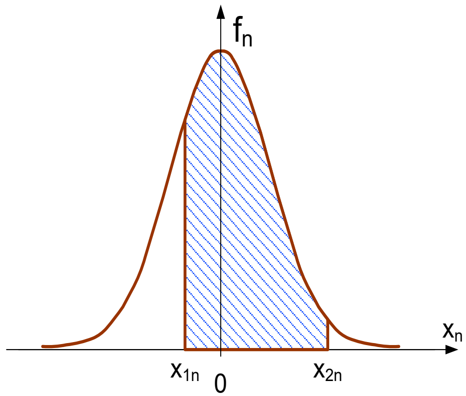
But for the N(0,1) distribution $F_n\left(-x_{1n}\right) = 1 - F_n\left(x_{1n}\right)$

therefore: $p = 2F_n\left(x_{1n}\right) - 1$

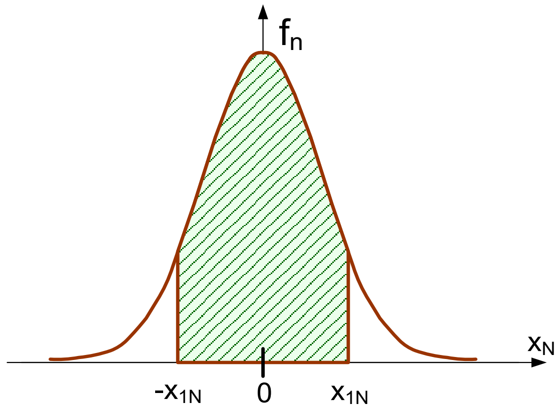


Background Information

$$p = F_n(x_{2n}) - F_n(x_{1n})$$



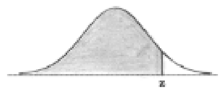
$$p = 2F_n(x_{1n}) - 1$$



Regardless of whether Gaussian performance requirements are asymmetric or symmetric, the CDF of the N(0,1) distribution (i.e. $F_n(x_n)$) can be used to characterize yield

Background Information

Tables of the CDF of the N(0,1) random variable are readily available. It is also available in Matlab, Excel, and a host of other sources.



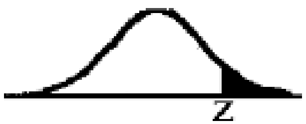
Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Background Information

Tables of the CDF of the N(0,1) random variable are readily available. It is also available in Matlab, Excel, and a host of other sources.



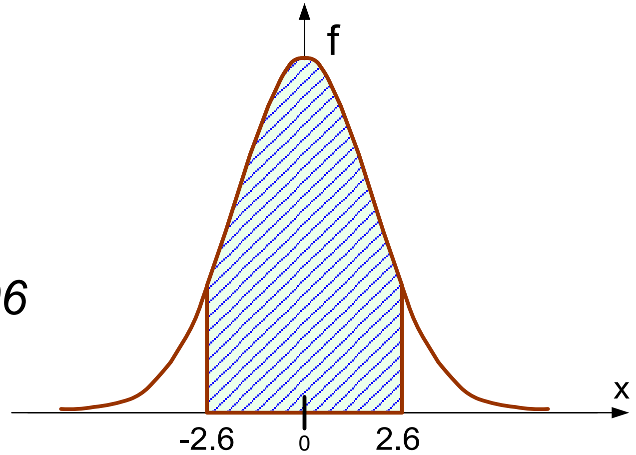
Far Right Tail Probabilities

Z	P{Z to oo}	Z	P{Z to oo}	Z	P{Z to oo}	Z	P{Z to oo}
2.0	0.02275	3.0	0.001350	4.0	0.00003167	5.0	2.867 E-7
2.1	0.01786	3.1	0.0009676	4.1	0.00002066	5.5	1.899 E-8
2.2	0.01390	3.2	0.0006871	4.2	0.00001335	6.0	9.866 E-10
2.3	0.01072	3.3	0.0004834	4.3	0.00000854	6.5	4.016 E-11
2.4	0.00820	3.4	0.0003369	4.4	0.000005413	7.0	1.280 E-12
2.5	0.00621	3.5	0.0002326	4.5	0.000003398	7.5	3.191 E-14
2.6	0.004661	3.6	0.0001591	4.6	0.000002112	8.0	6.221 E-16
2.7	0.003467	3.7	0.0001078	4.7	0.000001300	8.5	9.480 E-18
2.8	0.002555	3.8	0.00007235	4.8	7.933 E-7	9.0	1.129 E-19
2.9	0.001866	3.9	0.00004810	4.9	4.792 E-7	9.5	1.049 E-21

Background Information

Example: Determine the probability that the N(0,1) random variable has magnitude less than 2.6

$$p = 2F_n(2.6) - 1$$



From the table of the CDF, $F_n(2.6) = 0.9953$ so $p = .9906$

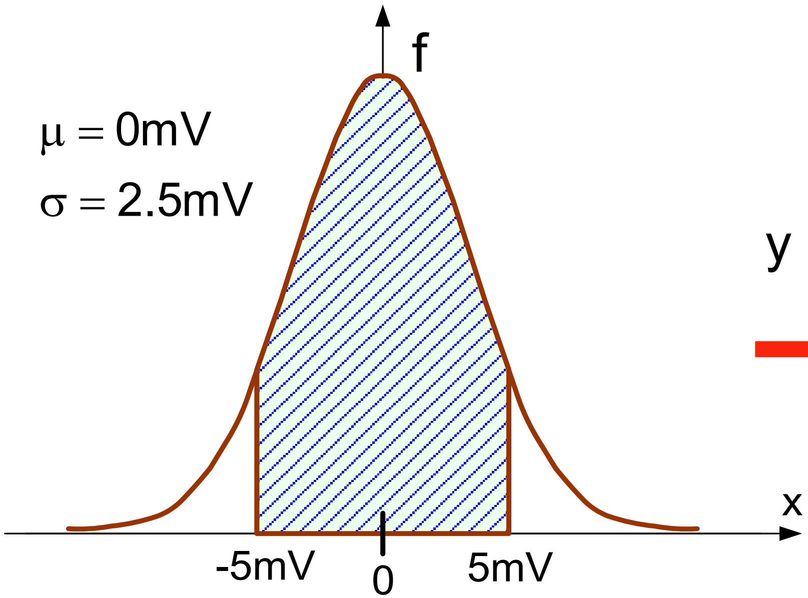
Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	[illegible]	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	[illegible]	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

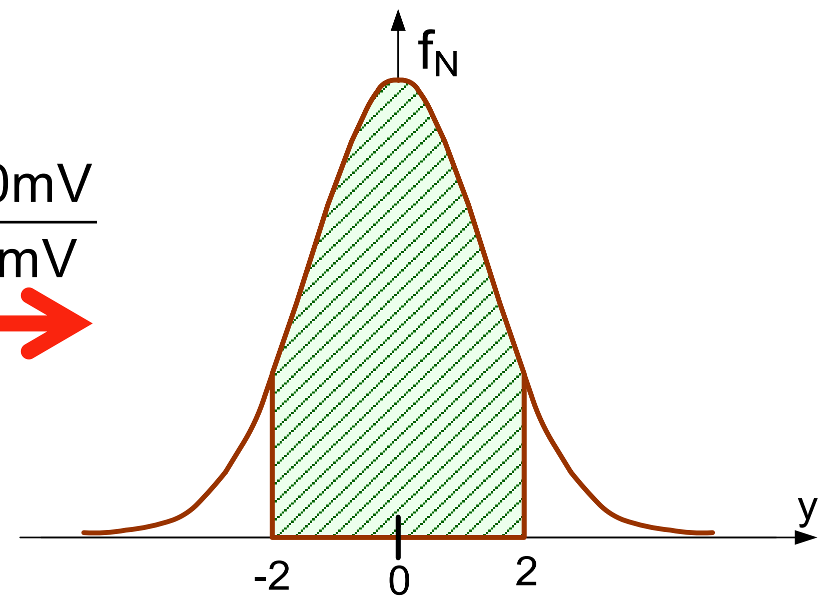
Background Information

It can be shown that the circuit designer has control of the offset voltage of an op amp and through architecture and sizing of devices can set the standard deviation of the offset voltage at almost any level. Invariably low offset voltages require larger area.

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.



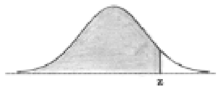
$$y = \frac{x - 0\text{mV}}{2.5\text{mV}}$$



$$p = \int_{-2}^{2} f_N(x)dx = F_N(2) - F_N(-2) = 2 * F_N(2) - 1$$

$$p = 2 * F_N(2) - 1$$

Example (continued)



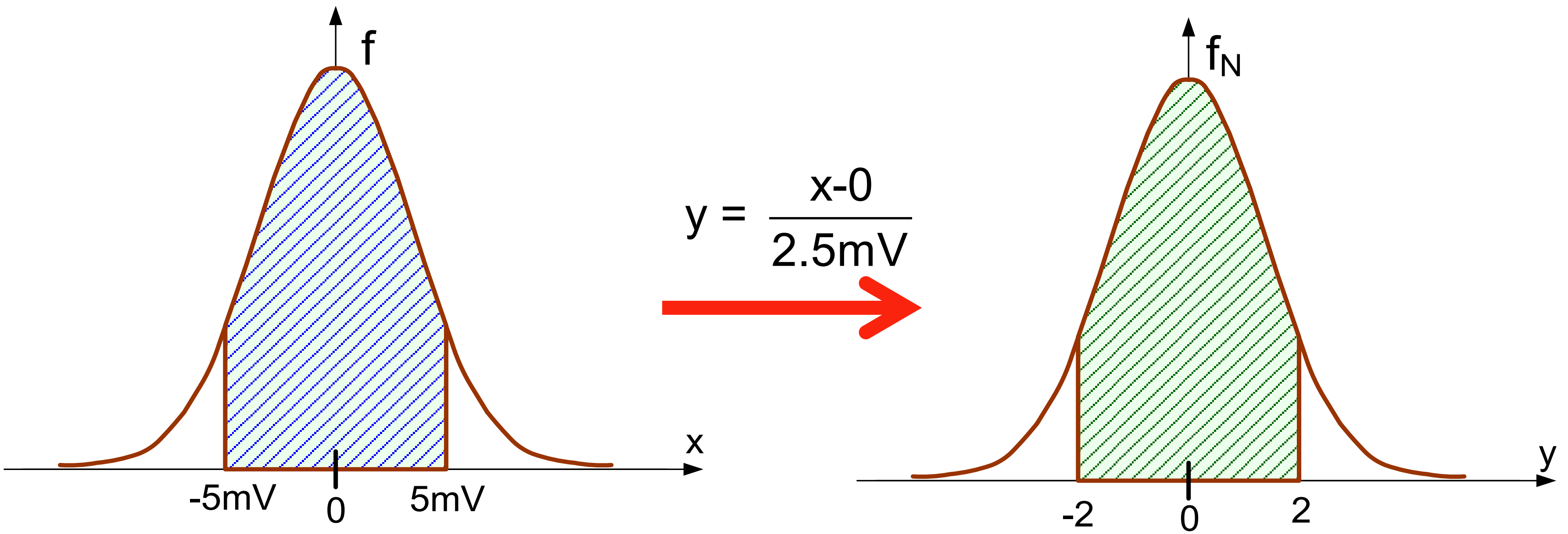
Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Example (continued)

Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 2.5mV and a mean of 0V.



$$p = 2 * F_N(2) - 1$$

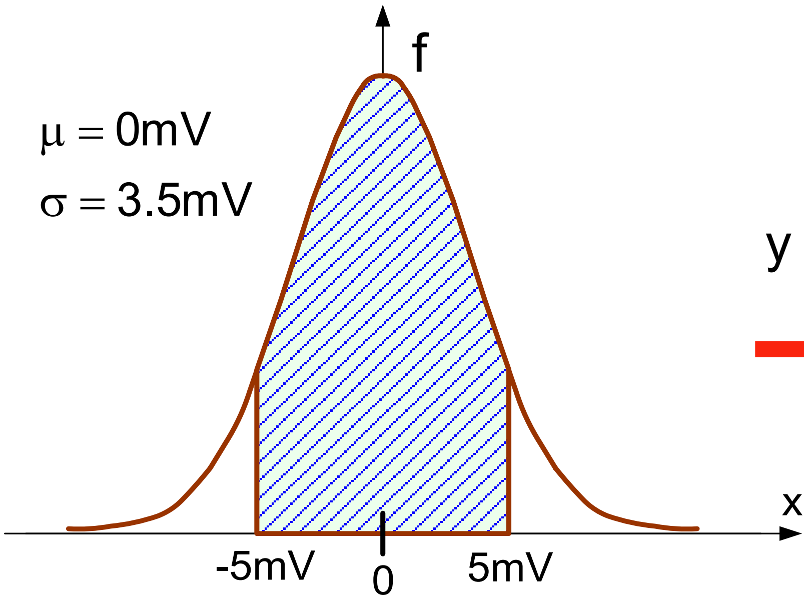
$$F_N(2) = 0.9772$$

$$p = 2 * .9772 - 1 = .9544$$

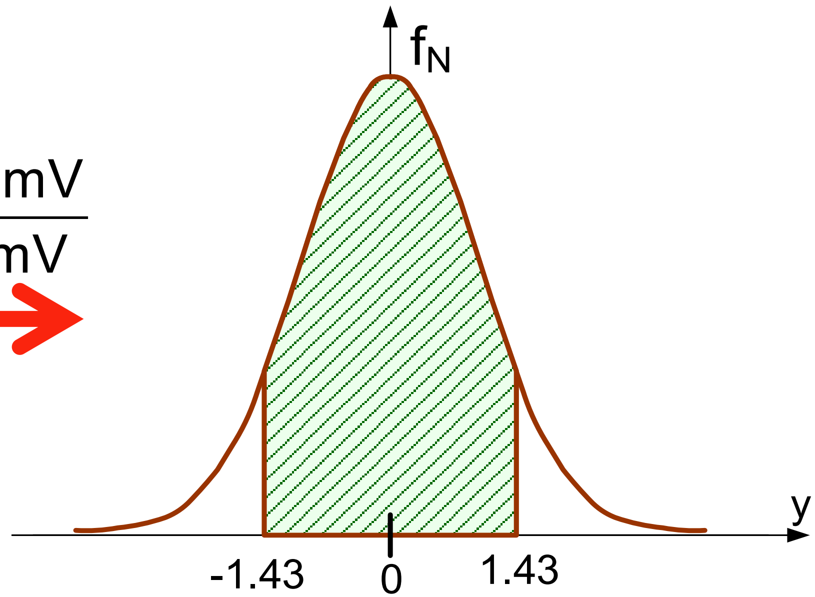
Background Information

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.



$$y = \frac{x - 0mV}{3.5mV}$$

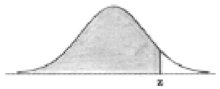


$$p = \int_{-1.43}^{1.43} f_N(x)dx = F_N(1.43) - F_N(-1.43) = 2 * F_N(1.43) - 1$$

$$p = 2 * F_N(1.43) - 1$$

Background Information

Example (continued)



Probability Content from -oo to Z

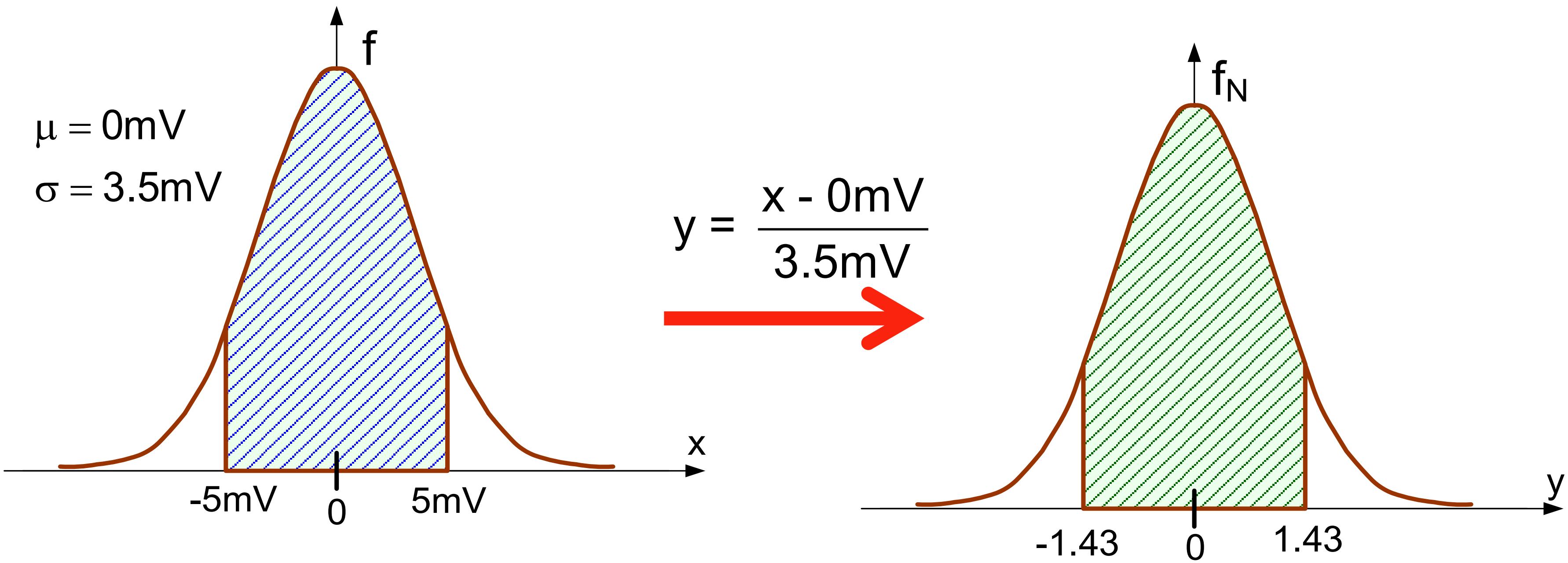
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

http://www.math.unb.ca/~knight/utility/NormTble.htm

Background Information

Repeat the previous example if the designer decided to reduce the area so that the standard deviation increased to 3.5 mV

Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the offset voltage has a Gaussian distribution with a standard deviation of 3.5mV and a mean of 0V.

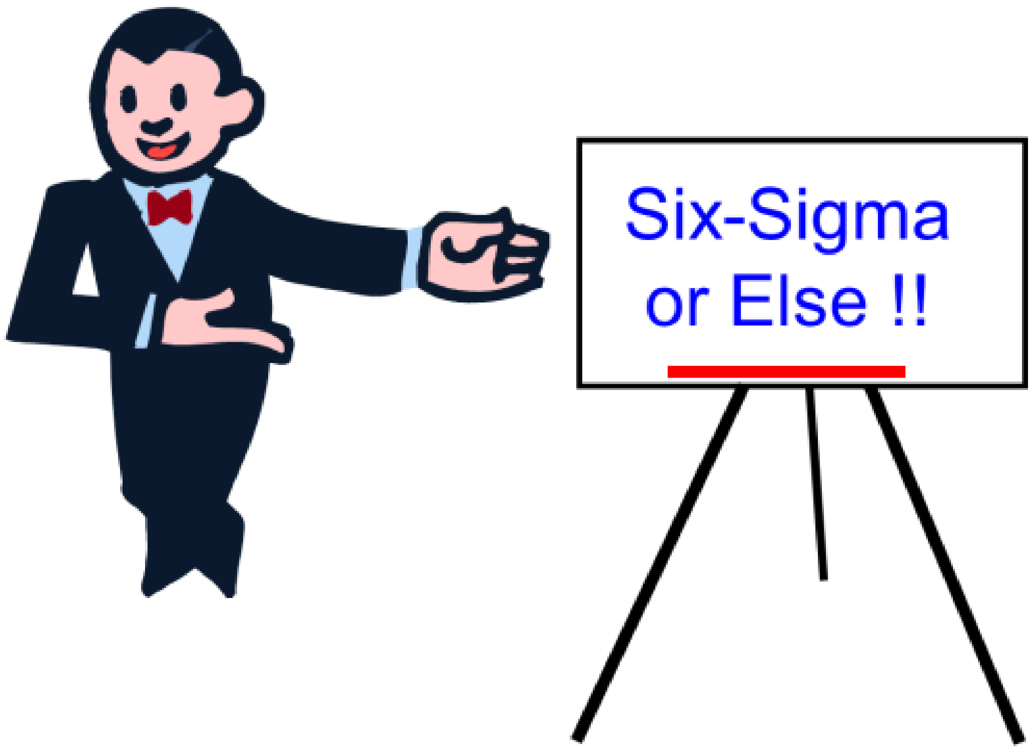


$$p = 2 * F_N(1.43) - 1 = 2 * 0.9236 - 1 = 0.847$$

This small change in the design dropped the yield from just over 95% to just under 85%

Statistical analysis is critical for predicting performance capabilities of many ICs !

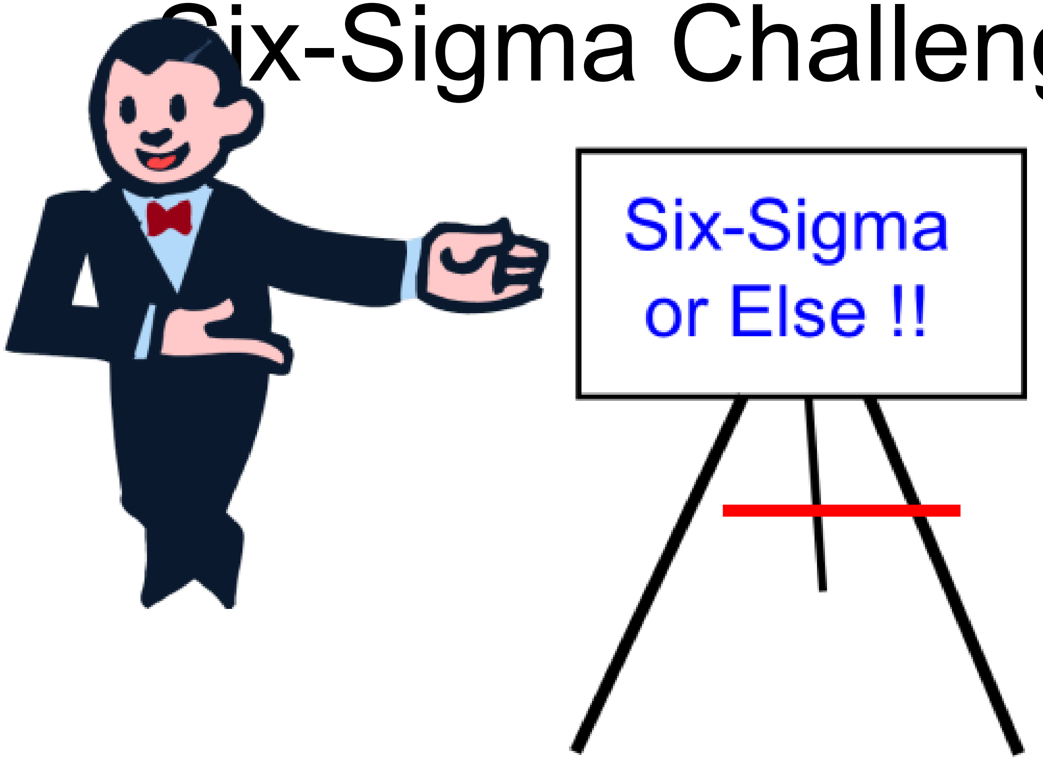
Many Companies Promote the Real Six-Sigma Challenge



From Wikipedia Sept 1 2021

Six Sigma (**6σ**) is a set of techniques and tools for process improvement. It was introduced by American engineer Bill Smith while working at Motorola in 1986.[1][2] A six sigma process is one in which 99.99966% of all opportunities to produce some feature of a part are statistically expected to be free of defects.

Many Companies Promote the Real Six-Sigma Challenge



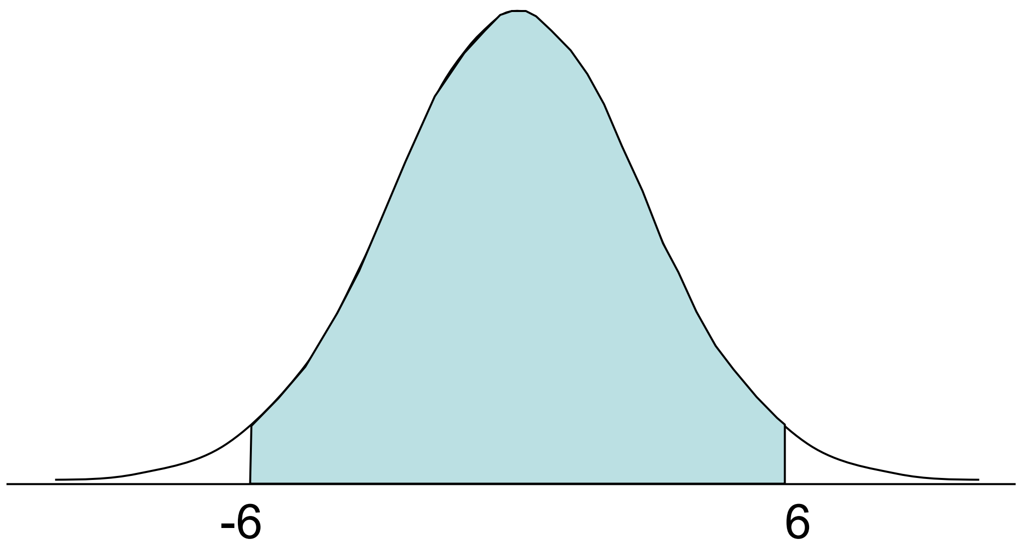
From Wikipedia   Sept 1 2021

In 2005 Motorola attributed over $17 billion in savings to Six Sigma.[3]

By the late 1990s, about two-thirds of the Fortune 500 organizations had begun Six Sigma initiatives with the aim of reducing costs and improving quality.[6]

Yield at the Six-Sigma level

(Assume a Gaussian distribution)



$$Y_{6sigma} = 2F_N(6) - 1$$

$$Y_{6sigma} = 0.9999999980$$

This is approximately 2 defects out of 1 billion parts

Stay Safe and Stay Healthy !

End of Lecture 4